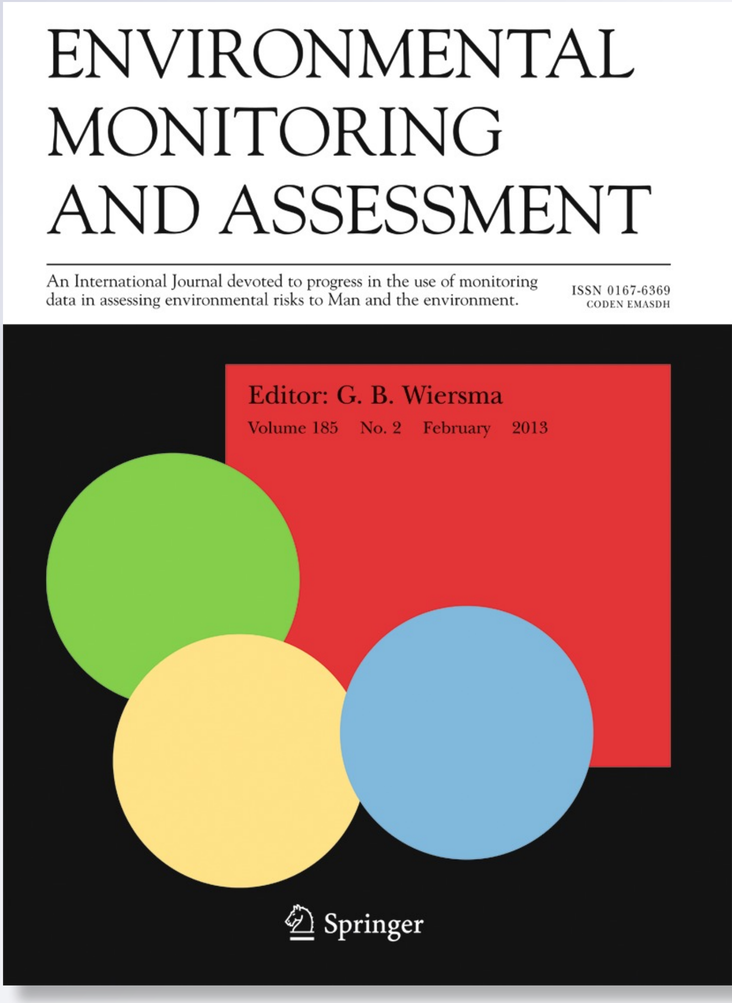
# *Rate of occurrence of failures based on a nonhomogeneous Poisson process: an ozone analyzer case study*

**José Carlos de Moura Xavier, Irany de Andrade Azevedo, Wilson Cabral de Sousa Junior & Augusto Nishikawa**

**Environmental Monitoring and Assessment**
An International Journal Devoted to Progress in the Use of Monitoring Data in Assessing Environmental Risks to Man and the Environment

ISSN 0167-6369
Volume 185
Number 2

Environ Monit Assess (2013)
185:1463-1471
DOI 10.1007/s10661-012-2645-6




Springer

Environ Monit Assess (2013) 185:1463–1471
DOI 10.1007/s10661-012-2645-6

# Rate of occurrence of failures based on a nonhomogeneous Poisson process: an ozone analyzer case study

**José Carlos de Moura Xavier · Irany de Andrade Azevedo · Wilson Cabral de Sousa Junior · Augusto Nishikawa**

Received: 16 August 2011 / Accepted: 2 April 2012 / Published online: 18 May 2012


**Abstract** Atmospheric pollutant monitoring constitutes a primordial activity in public policies concerning air quality. In São Paulo State, Brazil, the São Paulo State Environment Company (CETESB) maintains an automatic network which continuously monitors CO, $SO_2$, $NO_x$, $O_3$, and particulate matter concentrations in the air. The monitoring process accuracy is a fundamental condition for the actions to be taken by CETESB. As one of the support systems, a preventive maintenance program for the different analyzers used is part of the data quality strategy. Knowledge of the behavior of analyzer failure times could help optimize the program. To achieve this goal, the failure times of an ozone analyzer—considered a repairable system—were modeled by means of the nonhomogeneous Poisson process. The rate of occurrence of failures (ROCOF) was estimated for the intervals 0–70,800 h and 0–88,320 h, in which six and seven failures were observed, respectively. The results showed that the ROCOF estimate is influenced by the choice of the observation period, $t_0$=70,800 h and $t_7$=88,320 h in the cases analyzed. Identification of preventive maintenance actions, mainly when parts replacement occurs in the last interval of observation, is highlighted, justifying the alteration in the behavior of the inter-arrival times. The performance of a follow-up on each analyzer is recommended in order to record the impact of the performed preventive maintenance program on the enhancement of its useful life.

**Keywords** Repairable system · Rate of occurrence of failure · ROCOF · Reliability · Nonhomogeneous Poisson process · Ozone analyzer

J. C. de Moura Xavier (✉) · A. Nishikawa
CETESB—São Paulo State Environment Company,
Av. Prof. Frederico Hermann Júnior, 345,
CEP 05459-900 São Paulo, Brazil
e-mail: jxavier@sp.gov.br

A. Nishikawa
e-mail: auguston@cetesbnet.sp.gov.br

I. de Andrade Azevedo · W. C. de Sousa Junior
Aeronautics Institute of Technology (ITA),
Praça Marechal Eduardo Gomes, 50,
CEP 12228-900 São José dos Campos, Brazil

I. de Andrade Azevedo
e-mail: irany@ita.br

W. C. de Sousa Junior
e-mail: wilson@ita.br

**Notation**

| | |
|---|---|
| $t_i$ | $i$th system failure time (hours) |
| $t_0$ | End of observation interval (hours) |
| $n$ | Number of failures |
| $X_i$ | Inter-arrival time between $i$-1st and $i$th failures |
| $v(t)$ | Rate of occurrence of failures (ROCOF) at time $t$ |
| $E\{X\}$ | Expected value of random variable ($X$) |
| $m(t)$ | Time-dependent mean of the Poisson distribution |

| | |
|---|---|
| $N(t, t+\Delta t)$ | Number of failures in the interval $(t, t+\Delta t]$ |
| $\beta_i$ | Parameters of log-linear ROCOF |
| $\delta,\gamma$ | Parameters of power law process ROCOF |
| $\alpha$ | Significance level of the statistical test |
| $U$ | Laplace's test statistic |
| $R^2$ | Coefficient of determination |

## Introduction

Essential items to humans, other animals, and vegetal life, such as oxygen, carbon dioxide, nitrogen, and water, are present in the atmosphere and play a role in the cycles which are vital to this life. Some undesirable substances, which in general stem from anthropic activity, are also presented. These are principally generated by the burning of fossil fuels mainly to supply energy for different purposes, such as domestic heating, industrial activities, and automotive vehicles propulsion.

The World Health Organization (WHO) estimates over 2 million premature deaths per year, over 50 % of them in developing countries, attributed to external and internal atmospheric pollution caused by the burning of solid fuels (WHO 2002).

In the USA, adverse effects on public health and on urban aesthetics due to the intense process of industrialization brought about regulation for the control of emissions beginning in the late nineteenth century and extending to the present, with important landmarks, such as the Clean Air Act in 1963 which stipulated emission standards for automotive vehicles. Following that, amendments to the Act established some reference pollutants (criteria air pollutants) and emission standards: for sulfur dioxide ($SO_2$), ozone ($O_3$), nitric oxides ($NO_x$), carbon monoxide (CO), particulate matter (PM), and lead (Pb), pollutants which can cause adverse effects on human health and social welfare, limit concentrations were established focusing on protecting life (Álvares et al. 2002).

Pollutant monitoring has gained importance in medium- and large-sized urban centers all over the world. In Brazil, air quality standards and their respective reference methods were established as of 1990 by means of Resolution No. 03/90 of the National Council for the Environment (CONAMA).

In São Paulo State, CETESB, an organization associated with the Environment Secretariat, has been monitoring air quality since 1973. The first automatic station started operating in 1981, with electronic analyzers for $O_3$, $PM_{10}$, $NO_x$, CO, and $SO_2$ parameters. Systematic follow-up of pollutant behavior since then allows CETESB to establish and adjust public policy on air quality, proposing programs of emissions

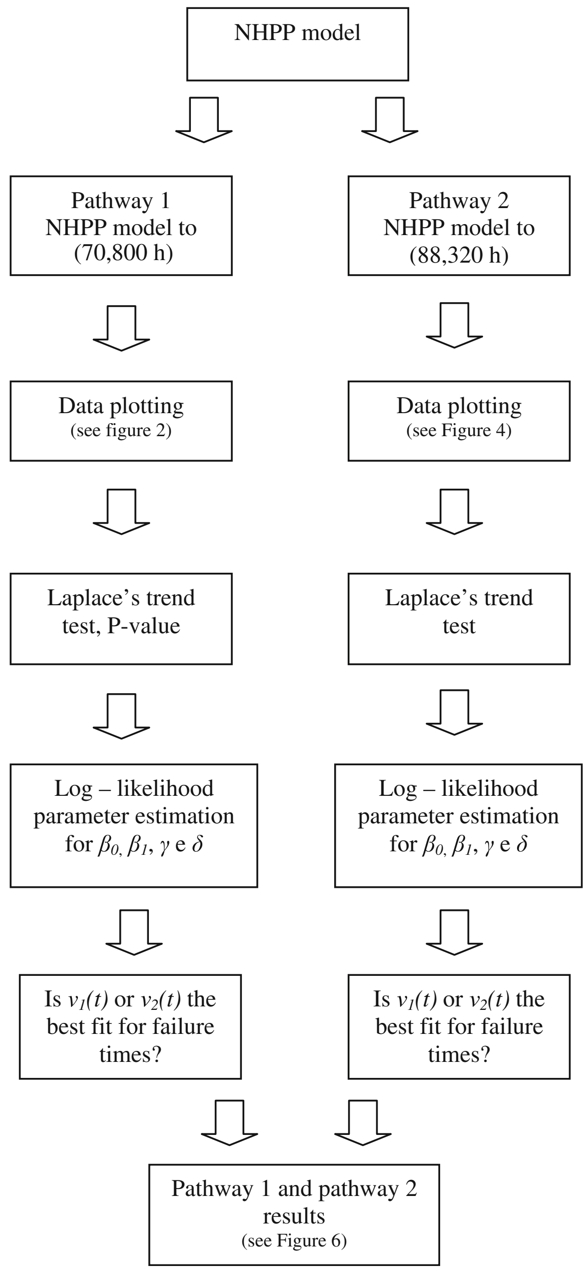



**Fig. 1** Pathways for analyzing the NHPP data

**Table 1** Failure times for ozone analyzer no. 1507

| Failure | Time to failure (h) | Inter-arrival time (h) |
|---|---|---|
| 1 | 23,208 | 23,208 |
| 2 | 37,824 | 14,616 |
| 3 | 47,472 | 9,648 |
| 4 | 63,192 | 15,720 |
| 5 | 66,984 | 3,792 |
| 6 | 68,208 | 1,224 |
| Sum $t_i$ | 306,888 | |

reduction at pollution sources as well as limitations on the licensing of new sources in regions where saturation by one or more pollutants has already or almost been reached.

Accuracy in the monitoring process is therefore a fundamental condition in assuring the quality of the measures taken by CETESB, being part of the data quality strategy, among others, a preventive maintenance program for the analyzers which was implemented at least 10 years ago.

Research aiming to optimize this program through knowledge of analyzer failures and prognoses of the behavioral trends of these failures is in progress, including the capability to estimate reliability and expected number of failures in upcoming years of operation. Furthermore, it is possible to evaluate whether the current maintenance program permits the optimizations associated with the periodicity in which the interruptions for maintenance and component replacements are performed, thus seeking to minimize costs and to maximize network availability.

In the present stage of the research, the nonhomogeneous Poisson process (NHPP) is used to model the behavior of the failure times of an ozone analyzer. The main goals were to quantify its rate of occurrence of failures (ROCOF), to observe its trends over time, and to determine how the observation period of failure times interferes in the estimation of ROCOF.

## Materials and methods

### Background

The theoretical basis for the reliability of repairable systems and the mathematical representation of their failure data can be found in Ascher and Feingold (1984), Cox and Lewis (1966), Crowder et al. (1991), and O'Connor (2002). The in-depth use of this conceptual basis for practical cases can be found in Ascher and Hansen (1998) and Saldanha et al. (2001).

A system is considered repairable when, after failing to satisfactorily perform at least one of its functions, it can be restored to a totally satisfactory performance by any method other than the replacement of the entire system (Ascher and Feingold 1984). Upon following up on this system over time, the essential parameter to be observed is the inter-arrival times, $X_i$, concerning maintenance instants where shorter intervals toward the end of the observation period may indicate system deterioration, while longer intervals could indicate a considerably extended durability (Ascher and Feingold 1984).

The idea of failures (localized events) occurring in a continuum (time), according to some probabilistic mechanism, agrees with the concept of the stochastic point process (Ascher and Feingold 1984; Cox and Lewis 1966; Crowder et al. 1991). Assuming that the repair time of a repairable system is usually shorter than the standby or operation times and that the time to

**Fig. 2** Cumulative number of failures versus cumulative operating time for ozone analyzer no. 1507

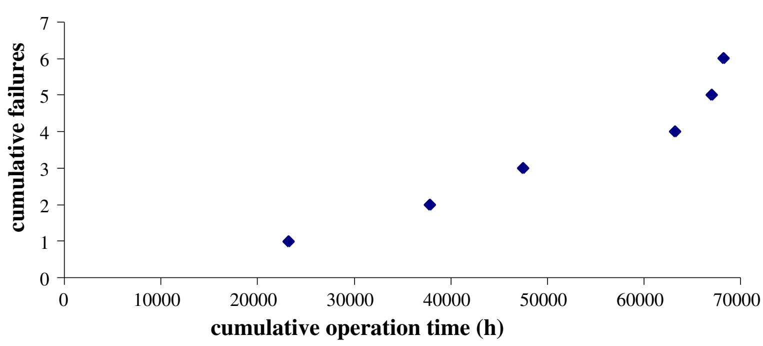


**Table 2** Data for the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameter estimations by graphic method and linear regression: sub-period 1

| Interval (h) | $n$ | $b_j$ | $v(b_j)\times 10^5$ | $\ln b_j$ | $\ln v(b_j)$ |
|---|---|---|---|---|---|
| 0–30,000 | 1 | 15,000 | 3.3 | 9.616 | −10.319 |
| 30,000–44,000 | 1 | 37,000 | 7.1 | 0.519 | −9.553 |
| 44,000–55,000 | 1 | 49,500 | 9.1 | 0.810 | −9.305 |
| 55,000–70,800 | 3 | 62,900 | 19.0 | 1.049 | −8.391 |

$R^2_{v1(t)}$=0.975; $R^2_{v2(t)}$=0.909

failure of any part is independent of any repair measures, stochastic point processes may be utilized, such as the Poisson process, to model the behavior of failure times and to estimate system reliability (O'Connor 2002), the Poisson process being a simple model having a well-developed statistical procedure of easy application (Crowder et al. 1991).

Although part of the recent literature claims that the stochastic point process is statistically adequate for the mathematical treatment of repairable systems, there are yet many mistakes concerning the use of probability distributions for repairable systems. Some authors point out that probability distributions are not adequate because they do not preserve the chronological sequence of inter-arrival times ($X_i$). Ascher and Feingold (1984) point out that, among other reasons, distributions are adequate only for the model behavior of each of their parts. They claim that the only period of lifetime to be considered must be the lifetime up to the first and only failure, the parts being discarded after that failure. A system commonly composed of parts, on the other hand, can be repaired and put into operation once again.

A stochastic point process is considered a homogeneous Poisson process (HPP) if it satisfies the following conditions: (1) $N(0)=0$; (2) $\{N(t),\ t\geq 0\}$ has independent and stationary increments; and (3) the number of events (in this case, failures) in each interval follows a Poisson distribution with an average $m(t)=vt$, with $0<v<\infty$, constant so that

$$P\{N(t,t+s]=k\}=\frac{(vt)^k\exp(-vt)}{k!} \tag{1}$$

for any $t$, $t\geq 0$ and $k=0, 1, 2, \ldots$.

A generalization of the HPP is the NHPP in which the $X_i$ is neither independent nor identically distributed, showing an average

$$m(t)=\int_0^t v(s)\mathrm{d}s \tag{2}$$

In this model, the ROCOF is defined as:

$$v(t)=\frac{d}{\mathrm{d}t}E\{N(t)\} \tag{3}$$

Crowder et al. (1991) report that a natural estimator for $v(t)$ when adequate $\Delta t$ are considered, the choice of intervals being arbitrary and decided by the user, can be

$$\widehat{v}(t)=\frac{N(t,t+\Delta t)}{\Delta t} \tag{4}$$

Saldanha et al. (2001) highlight that the NHPP allows one to consider events with a higher probability of occurrence in certain time intervals because it does not require the condition of having stationary time increments and that, due to its memory, it is a suitable tool to model events which undergo influences over time, such as aging.

When an increasing or decreasing trend in inter-arrival times ($X_i$) is observed, the NHPP model is recommended to represent the system's failures. A simple way to analyze that trend consists in plotting the cumulative number of failures against the cumulative time of these failures. The absence of linearity is a signal of increasing (concave monotonic growth) or decreasing (convex monotonic growth) ROCOF. Ascher and Feingold (1984) emphasized the existence

**Table 3** Data for the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameter estimations by graphic method and linear regression: sub-period 2

| Interval (h) | $n$ | $b_j$ | $v(b_j)\times 10^5$ | $\ln b_j$ | $\ln v(b_j)$ |
|---|---|---|---|---|---|
| 0–35,000 | 1 | 17,500 | 2.9 | 9.770 | −10.463 |
| 35,000–47,000 | 1 | 41,000 | 8.3 | 10.621 | −9.392 |
| 47,000–55,000 | 1 | 51,000 | 12.5 | 10.840 | −8.987 |
| 55,000–70,800 | 3 | 62,900 | 19.0 | 11.049 | −8.568 |

$R^2_{v1(t)}$=0.996; $R^2_{v2(t)}$=0.987

**Table 4** Data for the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameter estimations by graphic method and linear regression: sub-period 3

| Interval (h) | $n$ | $b_j$ | $v(b_j)\times 10^5$ | $\ln b_j$ | $\ln v(b_j)$ |
|---|---|---|---|---|---|
| 0–37,000 | 1 | 18,500 | 2.7 | 9.826 | −10.520 |
| 37,000–47,000 | 1 | 42,000 | 10.0 | 10.645 | −9.210 |
| 47,000–55,000 | 1 | 51,000 | 12.5 | 10.840 | −8.987 |
| 55,000–70,800 | 3 | 62,900 | 19.0 | 11.049 | −8.568 |

$R^2_{v1(t)}$=0.976; $R^2_{v2(t)}$=0.997

of simple statistical tools such the Laplace's test, as in Eq. 5. $U$ distribution tends to normal distribution, it being possible to test $U$ against the values of the variable $z$ of the standardized normal distribution. Considering the constant ROCOF (HPP model) as a null hypothesis ($H_0$)—therefore devoid of the $X_i$ trend—its rejection holds for $U>z_{\alpha/2}$, indicating an increasing ROCOF over time, or for $U<-z_{\alpha/2}$, indicating a decreasing ROCOF over time (Ascher and Feingold 1984).

$$U=\frac{\frac{\sum_{i=1}^{n} t_i}{n}-\frac{t_0}{2}}{t_0\sqrt{\frac{1}{12n}}} \qquad (5)$$

where $U$ is Laplace's statistic, $t_i$ is the time of the $i$th failure, $n$ is the number of failures, $t_0$ the end of observation interval.

The test undergoes a slight modification when the observation period ends on the $i$th failure. In this case, $\sum t_i$ must be replaced by $\sum t_{i-1}$, $n$ by $n-1$, and $t_0$ by $t_n$.

Rigdon and Basu (2000, p. 112) highlighted the importance of informing the $P$ value of the statistical test instead of merely accepting or refusing the null hypothesis $H_0$. The $P$ value is a probability and it informs the lowest level of significance which would lead to the rejection of $H_0$ based on the sample being analyzed. For tests based on the normal distribution, the $P$ value is obtained from the probability $P=2[1-\Phi(|u_0|)]$, a two-tailed test where $\Phi(|z|)$ represents the standardized normal distribution function. The $H_0$ is accepted if $\alpha<P$ value and rejected if $\alpha\geq P$ value (Montgomery and Runger 2003).

In engineering, an $s$-significance level of <5 % can usually be considered sufficient evidence upon which to reject $H_0$. A level of $s$-significance >10 % would not normally constitute sufficient evidence, and one could just as well reject the null hypothesis as perform more trials to obtain more data (O'Connor 2002, p.62).

The evidence of a trend in the ROCOF permits the application of the NHPP for the representation of the failure data of a repairable system. Cox and Lewis (1966) and Ascher and Feingold (1984) propose models for the ROCOF representation. The log-linear model (Crowder et al. 1991), originally proposed by Cox and Lewis (1966), and the power law process model (O'Connor 2002), presented in Eqs. 6 and 7, respectively, are initial options whose adequacy can be verified rapidly and do not imply in sophisticated calculations or complex mathematical models.

$$v_1(t)=\exp(\beta_0+\beta_1 t) \qquad (6)$$

$$v_2(t)=\gamma\delta t^{\delta-1} \qquad (7)$$

The estimation of the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameters can be accomplished by the method of maximum log-likelihood (Crowder et al. 1991). For a repairable system with the ROCOF expressed by $v_1(t)$ in the interval (0, $t_0$], with $n$ failures at $t_1$, $t_2$, $t_3$,…, $t_n$, the $\beta_1$ maximum log-likelihood estimator is obtained from

$$\sum_{i=1}^{n} t_i+n\beta_1^{-1}-nt_0\left\{1-e^{-\beta_1 t_0}\right\}^{-1}=0 \qquad (8)$$

and the $\beta_0$ estimator results from

**Table 5** Estimated parameters for the choice of $v(t)$

| Parameter | $\beta_1\times 10^5$ | $\beta_0$ | $\delta$ | $\gamma\times 10^{12}$ |
|---|---|---|---|---|
| Sub-period 1 | 3.50 | −10.87 | 2.098 | 376 |
| Sub-period 2 | 4.21 | −11.16 | 2.542 | 2.99 |
| Sub-period 3 | 4.43 | −11.25 | 2.440 | 21.40 |
| Log-likelihood | 4.32 | −11.27 | 2.555 | 2.43 |

**Table 6** Application of model $v_1(t)$ in the interval 0–70,800 h

| $t$ | $v(t)\times 10^5$ | $t$ | $v(t)\times 10^5$ | $t$ | $v(t)\times 10^5$ |
|---|---|---|---|---|---|
| 0 | 1.28 | 30,000 | 4.66 | 60,000 | 17.03 |
| 5,000 | 1.58 | 40,000 | 7.18 | 70,000 | 26.23 |
| 10,000 | 1.96 | 50,000 | 11.06 | 80,000 | 40.40 |
| 20,000 | 3.02 | | | | |

$$\widehat{\beta}_0 = \ln\left\{\frac{n\widehat{\beta}_1}{\exp\left(\widehat{\beta}_1 t_0\right) - 1}\right\} \tag{9}$$

If the observation interval ends with the $n$th failure, the $\beta_0$ and $\beta_1$ maximum log-likelihood estimators are obtained, replacing $t_0$ for $t_n$ in Eqs. 8 and 9 (Crowder et al. 1991, p. 166).

If the ROCOF is expressed by $v_2(t)$ in the interval $(0, t_0]$, with $n$ failures at $t_1, t_2, t_3, \ldots, t_n$, the $\delta$ and $\gamma$ maximum log-likelihood estimators are

$$\widehat{\delta} = \frac{n}{n\ln t_0 - \sum_{i=1}^{n}\ln t_i} \tag{10}$$

and

$$\widehat{\gamma} = \frac{n}{t_0^{\widehat{\delta}}} \tag{11}$$

If the observation interval ends with the $n$th failure, the $\delta$ and $\gamma$ maximum log-likelihood estimators are obtained from the substitution of $t_0$ for $t_n$ in Eqs. 10 and 11 (Rigdon and Basu 2000, p. 137). The question to be asked is: which of the two ROCOF models best fits the recorded failure data? Crowder et al. (1991) suggest the use of graphic methods to estimate the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameters and the comparison with those obtained from the log-likelihood method.

The use of the graphic method presumes the division of the observation interval $(0, t_0]$ into $k$ intervals $(0, t_1]$, $(t_1,t_2]$,…, $(t_{k-1},t_0]$ and the estimation of the $v(t)$ for each interval. Returning to Eq. 4, one estimate of the ROCOF is

$$\widehat{v}\left\{\frac{1}{2}\left(t_{j-1} + t_j\right)\right\} = \frac{N\left(t_j\right) - N\left(t_{j-1}\right)}{t_j - t_{j-1}} \tag{12}$$

The plotting of $\ln \widehat{v}\left(b_j\right)$ against $b_j = 1/2(t_{j-1} + t_j)$ should be fairly linear if $v_1(t)$ is adequate, supplying $\beta_0$ as a linear coefficient and $\beta_1$ as an angular coefficient of a straight line adjusted to the plotted points. The plotting of $\ln\widehat{v}\left(b_j\right)$ against $\ln b_j$ should be fairly linear if $v_2(t)$ is adequate, supplying the angular coefficient $\delta-1$ and the linear coefficient $\ln\gamma+\ln\delta$. If point adjustment is made by linear regression, the observation of coefficient of determination $R^2$ can assist in the choice of the number and size of the intervals. The choice of $k$ and $t_j$ is determined by the user, it being advisable to experiment with various subdivisions in order to verify that the visual impression given by the plotting is not heavily dependent on the grouping utilized.

## Equipment studied

The model 49B ozone analyzer, no. 1507, manufactured by Thermo Fisher Scientific Inc., USA, is an ultraviolet photometer which measures pollutant concentrations through attenuation of ultraviolet light due to the presence of ozone within the absorption cell. It is comparable to the equivalent no. EQOA-0880-047

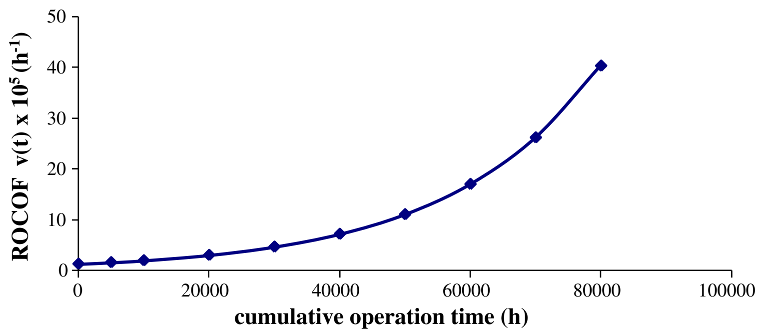



**Fig. 3** ROCOF behavior in the period 0–70,800 h for ozone analyzer no. 1507

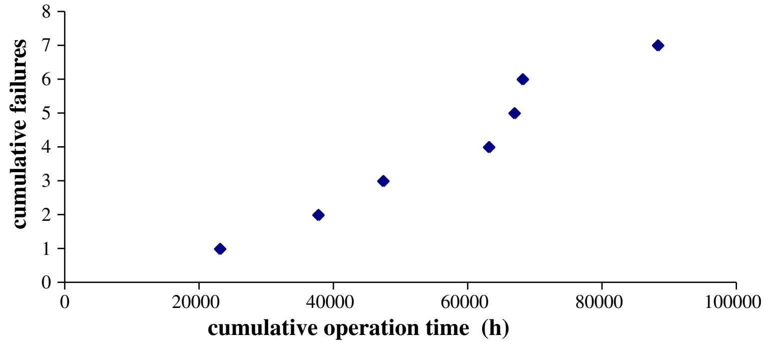



**Fig. 4** Cumulative number of failures versus time up to the seventh failure for ozone analyzer no. 1507

USEPA method. Its operation began on 12 January 1998. Six shutdowns have been reported, involving replacement of faulty components, the last register (68,208 h) occurring in 2006. Two additional shutdowns for preventive maintenance were registered, extending the operation time of the analyzer to 88,320 h (31 December 2008), without any further failures having been reported.

### Analysis strategy

The use of the nonhomogeneous Poisson process as a model to represent the behavior of the analyzer failure times—a repairable system—during its operational life has been developed along the two pathways shown in Fig. 1. The first pathway considers the end of the observation period to be 31 December 2006, with the duration corresponding to $t_0$=70,800 h. This choice corresponds to the hypothesis of applying this model a few months after the occurrence of the sixth and final failure. The second pathway considers the hypothetical occurrence of the seventh failure at $t_7$=88,320 h.

As part of the development of both pathways, the necessity for the correct interpretation of the result when the ROCOF is estimated after a long interval without failures, in which the preventive maintenance was performed, becomes clear.

## Results and discussion

Pathway 1 initiates with the graphic representation of the cumulative number of failures and the respective observation interval (0–70,800 h). Table 1 shows the failure times chronologically ordered up to $t_6$, the interval between these failures and the sum of failure times up to $t_6$. Figure 2 shows the cumulative number of failures and respective cumulative operation time, with a concave monotonic inclination indicating a crescent ROCOF over time.

As the observation interval ends at $t_0$, the Laplace's test considers $\sum t_i$=306,888 h, $n$=6, and $t_0$=70,800 h. The figure obtained is $U$=1.89, which proves sufficient evidence against $H_0$ (absence of trend) at the 10 % level of $s$-significance. The $P$ value of the test is 0.059, less than the significance level $\alpha$=0.1, which ratifies the decision to reject the hypothesis $H_0$ of the constant ROCOF.

Representation of ROCOF can be obtained from Eq. 6 or Eq. 7, the next step being the estimation of their $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameters, adjusting the failure data by means of the log-likelihood model. Using the log-linear model, Eqs. 6, 8, and 9 are applied, obtaining $\widehat{\beta} = -11.27$ and $\widehat{\beta}_1 = 4.32 \times 10^{-5}$. Thus,

$$v_1(t) = \exp(-11.27 + 4.32 \times 10^{-5} t) \tag{13}$$

**Table 7** Estimated parameters for the choice of $v(t)$ considering the seventh failure

| Parameter | $\beta_1 \times 10^5$ | $\beta_0$ | $\delta$ | $\gamma \times 10^8$ |
|---|---|---|---|---|
| Sub-period 1 | 1.92 | −10.40 | 1.639 | 5.29 |
| Sub-period 2 | 1.93 | −10.41 | 1.641 | 5.18 |
| Sub-period 3 | 1.99 | −10.44 | 1.685 | 3.16 |
| Log-likelihood | 1.99 | −10.44 | 1.905 | 0.26 |

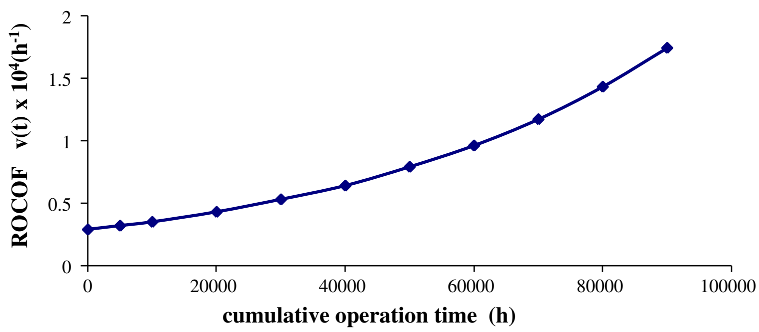



**Fig. 5** ROCOF behavior in the period 0–88,320 h for ozone analyzer no. 1507

For the power law process model (Eq. 7), Eqs. 10 and 11 are applied, obtaining $\widehat{\gamma} = 2.43 \times 10^{-12}$ and $\widehat{\delta} = 2.555$. Thus,

$$v_2(t) = 6.22 \times 10^{-12} t^{1.555} \qquad (14)$$

The ROCOF obtained from maximum log-likelihood estimators for the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameters can be expressed by Eq. 13 or Eq. 14. The choice between them to verify which best fits the data in Table 1 was made by graphic method and linear regression based on three sub-periods, indicated in Tables 2, 3, and 4.

Table 5 shows the estimated parameters for the linear regression, considering sub-periods 1–3 and the estimates for the log-likelihood. Considering the values for sub-period 3, it is understood that the model $v_1(t)$ best suits the data in Table 1. Table 6 shows the use of the model $v_1(t)=\exp(-11.27+4.32\times 10^{-5}t)$ in the interval 0–70,800 h. Figure 3 shows the behavior of $v_1(t)$ in the interval 0–70,800 h.

Failures in ozone analyzers were not reported in 2007 and 2008, and the occurring interruptions were due to preventive maintenance. Pathway 2 in Fig. 1 considers the hypothetical occurrence of the seventh failure at $t$=88,320 h, a plausible situation once the equipment had been in operation up to that moment.

Aggregating the seventh (hypothetical) failure to Table 1, Laplace's test parameters turn into $\sum t_{i-1}$= 306,888 h, $n-1$=6, and $t_7$=88,320 h. The $U$=0.67 result is not large enough to reject $H_0$ (absence of trend) at the 10 % level of $s$-significance. Nevertheless, considering the strong evidence that the observed prolonging of the useful life of the system was due to the preventive maintenance practiced, the steps for the modeling of a NHPP were repeated, despite there being no possibility of discarding $H_0$, with an $s$-significance of up to 10 %.

Figure 4 suggests the occurrence of a ROCOF trend change, made evident by the $X_7$ value, when compared to $X_6$ or $X_5$. Although Crowder et al. (1991) informed the existence of models which combined $v_1(t)$ and $v_2(t)$, in such a way as to better represent the observed inversion of the (unique) trend, this alternative was not considered in the present study.

The step repetitions in the $\beta_0$, $\beta_1$, $\gamma$, and $\delta$ parameter estimations by log-likelihood for the period ending at $t_7$ and the choice of $v_1(t)$ or $v_2(t)$ as the best representation of failure times are synthesized in Table 7. Considering the values for sub-period 3, it is understood that the

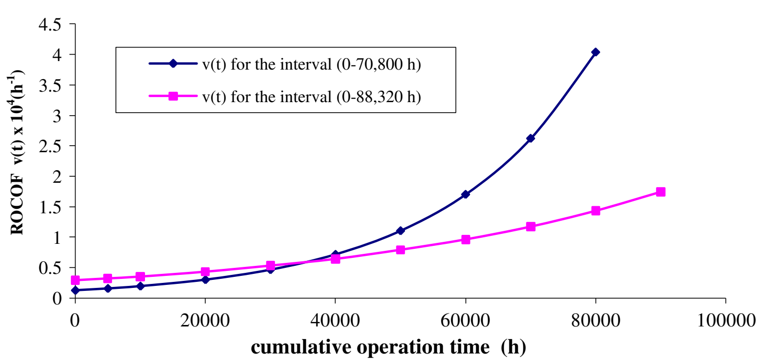



**Fig. 6** Comparison of the ROCOFs in the periods 0–70,800 h and 0–88,320 h for ozone analyzer no. 1507

model $v_1(t)$ best suits the data in Table 1, when the seventh failure is included.

Figure 5 shows the behavior of $v_1(t)=\exp(-10.44+1.99\times10^{-5}t)$ when the interval 0–88,320 h is considered. Figure 6 shows the ROCOF behavior in the recorded observation periods. Upon considering the ROCOF trend in the period 0–70,800 h, one might infer the occurrence of an increasing number of failures in intervals similar to $X_5$ or $X_6$. The insertion of the seventh failure in the analysis alters the ROCOF trend, permitting one to infer that the maintenance procedures had been responsible for that.

The aforementioned technical background and strategy were also employed to analyze the failure data of the $SO_2$, $NO_x$, CO, and PM monitors. The results of both ROCOFs and the expected number of failures can be found in Xavier (2011).

## Conclusions

The results seem to endorse that the estimation of the rate of occurrence of failures for a repairable system is influenced by the choice of the observation period. Between $t_6$=68,208 h and the hypothetical failure at $t_7$=88,320 h, two preventive maintenances were performed, with part substitutions at $t$=70,992 h and $t$=80,736 h, and the inversion of the concavity between $X_6$ and $X_7$, shown in Fig. 4, may be accountable for the preventive maintenance considered in $X_7$.

The flatter ROCOF behavior in interval 0–88,320 h was influenced by the inversion of the trend in the intervals between failures ($X_i$), with $X_7>>X_6$. Information concerning the occurrence of the preventive maintenance procedures, with the replacement of parts, mainly during the last observation interval, was shown to be essential for the correct interpretation of the ROCOF when it is estimated after a long interval without failures.

In order to have a better understanding of both the observed trend and its association with the preventive maintenance procedures practiced, one should perform a follow-up on the analyzer for at least a period identical to the $X_7$, with due registers of part failures and substitutions.

Failures occurring during the active analyzer life cycle, if in significant number, decrease the availability of measured data, with a possible interference in CETESB's policy for environment. To minimize the number of failures, it is necessary to know the causes and their mechanisms as well as the trend for growth over time. From this diagnosis, one can then propose measures to improve the maintenance programs for this equipment, with an expected improvement in its reliability and availability.

## References


Álvares, O. M., Jr., Lacava, C. I. V., & Fernandes, P. S. (2002). *Emissões atmosféricas*. Brasília: SENAI/DN.

Ascher, H. E., & Feingold, H. (1984). *Repairable systems reliability, modeling, inference, misconceptions and their causes*. New York: Marcel Dekker.

Ascher, H. E., & Hansen, C. K. (1998). Spurius exponentiality observed when incorrectly fitting a distribution to nonstationary data. *IEEE Transactions on Reliability, 47*(04), 451–459.

Cox, D. R., & Lewis, P. A. W. (1966). *The statistical analysis of series of events*. London: Latimer Trend.

Crowder, M. J., Kimber, A. C., Smith, R. L., & Sweeting, T. J. (1991). *Statistical analysis of reliability data*. Boca Raton: Chapman & Hall.

Montgomery, D. C., & Runger, G. C. (2003). *Applied statistics and probability for engineers* (3rd ed.). New York: Wiley.

O'Connor, P. D. T. (2002). *Practical reliability engineering* (4th ed.). New York: Wiley.

Rigdon, S. E., & Basu, A. P. (2000). *Statistical methods for the reliability of repairable systems*. New York: Wiley.

Saldanha, P. L. C., de Simone, E. A., & Frutuoso e Melo, P. F. (2001). An application of non-homogeneous Poisson point processes to the reliability analysis of service water pumps. *Nuclear Engineering and Design, 210*, 125–133.

World Health Organization (WHO) (2002). *The World Health Report 2002*: reducing risks, promoting healthy life. Switzerland: WHO Press. http://www.who.int/whr/2002/em/whr02_en.pdf. Accessed 12 May 2011.

Xavier, J. C. de M. (2011). *Análise da disponibilidade da rede automática de monitoramento da qualidade do ar e seus reflexos no licenciamento ambiental realizado em São Paulo*. 123f. Master dissertation. Brasil: Instituto Tecnológico de Aeronáutica. Retrieved 21 March 2012 from http://www.bd.bibl.ita.br.